

**APPLICATION OF NUMERICAL OPTIMIZATION TECHNIQUES TO CONTROL
SYSTEM DESIGN FOR NONLINEAR DYNAMIC MODELS OF AIRCRAFT**

C. Edward Lan and Fuying Ge
Department of Aerospace Engineering
The University of Kansas, Lawrence, Kansas 66045

ABSTRACT

Control system design for general nonlinear flight dynamic models is considered through numerical simulation. The design is accomplished through a numerical optimizer coupled with analysis of flight dynamic equations. In the analysis, the general flight dynamic equations are numerically integrated and dynamic characteristics are then identified from the dynamic response. The design variables are determined iteratively by the optimizer to optimize a prescribed objective function which is related to desired dynamic characteristics. Generality of the method allows nonlinear effects of aerodynamics and dynamic coupling to be considered in the design process. To demonstrate the method, nonlinear simulation models for an F-5A and an F-16 configurations are used to design dampers to satisfy specifications on flying qualities and control systems to prevent departure. The results indicate that the present method is simple in formulation and effective in satisfying the design objectives.

INTRODUCTION

At high angles of attack, the aerodynamic forces and moments are, in general, time-dependent and nonlinear functions of motion variables. Therefore, the traditional control system design method based on a linearized dynamic system are not appropriate. In addition, the aerodynamic, kinematic, and inertial coupling phenomena are important to the high angle-of-attack flight dynamics of modern aircraft. As a result, a number of high angle-of-attack control concepts have emerged (refs. 1-4). Therefore, a suitable control system design method must be capable of incorporating these coupling phenomena with considerations of time-dependent, nonlinear aerodynamic forces and moments. A control system designed without considering these coupling phenomena often has a detrimental effect on the departure/spin resistance (ref. 5). Another feature of high-alpha control system is the simultaneous utilization of several control surfaces or devices. Therefore, a design method capable of handling multiple input and output is necessary. A current approach to solving this problem is by extensive piloted simulation (ref. 5).

Methods in optimal control theory represent possible approaches to solving these problems under consideration. These methods are derived through calculus of variation. However, computational methods in existence require linearization of dynamic equations and aerodynamics about trimmed conditions (ref. 6). Another alternative is to apply numerical optimization techniques without linearization as they are frequently used in structural and aerodynamic designs of large systems. A similar approach has also been used in other control applications in ref. 7.

In the present method, a numerical optimization technique based on conjugate gradients and feasible directions (ref. 8) is coupled with an analysis method which is to obtain numerical solutions of the nonlinear six degree-of-freedom dynamic equations. This analysis method is to provide information needed in the design process, such as damping ratios, frequencies, motion variables involved in dynamic instabilities, etc. Since the analysis method can deal with nonlinearities in the dynamics and the aerodynamics and with any general constraints on the control system configuration, the control system designed with a numerical optimization technique can be very realistic and effective.

NUMERICAL APPROACHES

Typically, a control system may be designed to enhance flying qualities, to prevent flight departure, and to have an effective maneuver control system. To demonstrate the present method, only the first two objectives will be considered. That is, one is to design dampers at a moderate angle of attack to satisfy specifications on flying qualities and the other to design a control system to prevent flight departure at high angles of attack in a maneuver. Numerical formulations to solve these problems are described in the following.

Design to Satisfy Flying Qualities Specifications

The general system of equations can be written as

$$m(\dot{u} - vr + wq) = mg_x + F_{A_x} + F_{T_x} \quad (1a)$$

$$m(\dot{v} + ur - wp) = mg_y + F_{A_y} + F_{T_y} \quad (1b)$$

$$m(\dot{w} - uq + vp) = mg_z + F_{A_z} + F_{T_z} \quad (1c)$$

$$I_{xx}\dot{p} - I_{xz}\dot{r} - I_{xz}pq + (I_{zz} - I_{yy})rq = L_A + L_T \quad (1d)$$

$$I_{yy}\dot{q} + (I_{xx} - I_{zz})pr + I_{xz}(p^2 - r^2) = M_A + M_T \quad (1e)$$

$$I_{zz}\dot{r} - I_{xz}\dot{p} + (I_{yy} - I_{xx})pq + I_{xz}qr = N_A + N_T \quad (1f)$$

$$\dot{\phi} = p + q \sin\phi \tan\theta + r \cos\phi \tan\theta \quad (1g)$$

$$\dot{\theta} = q \cos\phi - r \sin\phi \quad (1h)$$

$$\dot{\psi} = (q \sin\phi + r \cos\phi)\sec\theta \quad (1i)$$

$$\alpha = \tan^{-1}(w/u) \quad (1j)$$

$$\beta = \sin^{-1}(v/\sqrt{u^2 + v^2 + w^2}) \quad (1k)$$

where (u, v, w) are the three linear velocity components of the aircraft; (p, q, r) are the angular velocity components; and (ϕ, θ, ψ) are the Euler angles in bank, pitch, and yaw, respectively. The subscripts (x, y, z) appearing on the right-hand side of Eqs. (1a) - (1c) denote components in the corresponding coordinate directions; and (A, T) denote aerodynamic and thrust components, respectively. g is the gravitational acceleration, and F 's are the external forces, while (L, M, N) are the moments about the $(x-, y-, z-)$ axes. In addition, m is the mass and I_{xx}, I_{xz} , etc., are the moments of inertia. The aerodynamic forces and moments (F_A, L_A, M_A, N_A) , including the control effects, are represented in dimensionless coefficients in a tabulated form as functions of motion variables in this study. The motion variables are (u, v, w, p, q, r) .

This system of equations is numerically integrated from an initial state (usually a trimmed level flight condition) after disturbances (such as impulsive control-surface deflections) are imposed to generate time-history data of motion variables.

For demonstrative purposes, it is assumed that dampers to provide flight characteristics satisfying flying-qualities specifications are to be determined. This problem has been solved in the past by conventional methods, such as the root-locus method, by using linearized equations of motion. It is considered here mainly to show the generality of the present method even without linearizing the equations of motion. In the present method, the necessary design information includes damping ratios, natural frequencies, and time constants of the vehicle motions. These characteristics are identified from calculated time-history results of motion variables after multiple-axis disturbances are imposed. The numerical method used for parameter identification is the method of differential corrections described in the following.

A general discretized system output in the time domain is assumed to be of the form:

$$f(t_k) = \sum_{i=1}^n e^{-\zeta_i \omega_{n_i} t_k} (A_i \cos \omega_i t_k + B_i \sin \omega_i t_k) + \sum_{j=1}^m C_j e^{-\sigma_j t_k} + D t_k + E \quad (2)$$

where $t_k = \Delta t(k - 1)$, $k = 1, 2, \dots, K$, $\omega_i = \omega_{n_i} \sqrt{1 - \zeta_i^2}$ is the damped frequency of the i^{th} mode. The objective is to use Eq. (2) to fit the dynamic response data $[Q_k = x(t_k)]$ through the method of least squares to determine the damping ratios (ζ_i) natural frequencies (ω_{n_i}) and time constants $(1/\sigma_j)$, $i = 1, \dots, n$;

$j = 1, \dots, m$. These parameters appear nonlinearly in Eq. (2). Other unknowns, A_i, B_i, C_j, D , and E , are linear parameters in Eq. (2). Because of nonlinearity, finding a solution of the resulting nonlinear algebraic equations from the least-square formulation is difficult. The best approach, as it has been determined in the present investigation, is the method of differential corrections. In other words, the unknown parameters are expressed as

$$\zeta_i = \zeta_{i_k} + \Delta\zeta_i$$

$$\omega_{n_i} = \omega_{n_{i_k}} + \Delta\omega_{n_i}, \text{ etc.}, i = 1, \dots, n, j = 1, \dots, m$$

where ζ_{i_k} , $\omega_{n_{i_k}}$, ... are the initial approximations of the i^{th} or j^{th} unknowns. Using Taylor series expansions, it is obtained that

$$\begin{aligned} Q_k + \varepsilon_k = f_o(t_k) + \sum_{i=1}^n (\Delta\zeta_i \frac{\partial f}{\partial \zeta_{i_k}} + \Delta\omega_{n_i} \frac{\partial f}{\partial \omega_{n_{i_k}}} + \Delta A_i \frac{\partial f}{\partial A_{i_k}} + \Delta B_i \frac{\partial f}{\partial B_{i_k}}) \\ + \sum_{j=1}^m (\Delta C_j \frac{\partial f}{\partial C_{j_k}} + \Delta\sigma_j \frac{\partial f}{\partial \sigma_{j_k}}) + \Delta D \frac{\partial f}{\partial D_k} + \Delta E \frac{\partial f}{\partial E_k} + \dots \end{aligned} \quad (3)$$

where ε_k is the residual. The least-square method is then applied to Eq. (3) in such a way that

$$G = \sum_{k=1}^K \varepsilon_k^2 = \text{minimum}$$

The differential corrections ($\Delta\zeta_i$, etc.) which minimize the G-function are determined by setting the first derivatives, $\partial G / \partial (\Delta\zeta_i)$, etc., to zero. Once the differential corrections are determined, they are added to the initial estimates of the unknowns and the process is repeated to determine a new set of differential corrections until G is a minimum or until there is no significant change in the unknowns. Typically, convergence is assumed if $G < 10^{-7}$.

After the necessary design information is obtained from the analysis part of the algorithm, the optimizer is called to perform the design process.

The damper design problem here may be formulated as follows: find the pitch rate feedback gain K_q , the roll rate feedback gain K_p , the yaw rate feedback gain K_r , the lateral acceleration feedback gain K_{ay} , and the aileron-to-rudder interconnect gain K_{ARI} , such that the following objective function is minimized:

$$\begin{aligned} OBJ = \frac{-B1}{\varepsilon + E1 \times |\zeta_{sp_1} - \zeta_{sp}|} + \frac{-B2}{\varepsilon + E2 \times |\zeta_{p_1} - \zeta_p|} \\ + \frac{-B3}{\varepsilon + E3 \times |\zeta_{D_1} - \zeta_D|} + \frac{-B4}{\varepsilon + E4 \times |\omega_{n_{D_1}} - \omega_{n_D}|} \\ + \frac{-B5}{\varepsilon + E5 \times |T_{r_1} - T_r|} + \frac{-B6}{\varepsilon + E6 \times |T_{s_1} - T_s|} \end{aligned} \quad (4)$$

where ζ_{sp_1} , ζ_{p_1} , ζ_{D_1} , $\omega_{n_{D_1}}$, T_{r_1} , and T_{s_1} are specified values to satisfy MIL-

F-8785B. ζ_{sp} , ζ_p , ζ_D , ω_{n_D} , T_t , and T_s are values obtained in the analysis

part. In Eq. (4), B_i and E_i are some weighting factors. ϵ in the denominator of the objective function is a small number used to prevent the objective function from being infinite and is set to 10^{-14} in the present algorithm. The optimization problem is subject to constraints on magnitudes of damping ratios, frequencies, time constants, overshoot, etc. In the optimization process, the design variables are varied systematically by the optimizer to obtain numerically the gradients of the objective function and constraints. These gradients are then used through the methods of conjugate gradients and feasible directions to determine the appropriate design variables to minimize the objective function. The process continues until the objective function does not change and the constraints are all satisfied.

Design to Prevent Flight Departure

Again, Eqs. (1) are numerically integrated. During time integration, a certain maneuver flight is imposed to induce departure of the airplane. One example of the maneuver flight is to pull up the airplane (i.e., to increase the angle of attack) and then induce a high roll rate afterwards. The present algorithm is constructed on the assumption that a departure condition is identifiable from the magnitude of the state vector, or motion variables. Since the latter are directly obtained from time integration of Eqs. (1), no further data manipulation is needed to calculate the necessary design information.

The design objective is achieved by first assuming a control system structure. Then the design problem may be formulated for the demonstration cases to be presented as follows.

Determine the aileron-rudder interconnect gain (K_{ARI}), the side acceleration feedback gain (K_{ay}), and the yaw damper gain (K_r), etc., to

minimize the following objective function:

$$\begin{aligned} \text{OBJ} = & -C_1 p_{\max} - C_2 \alpha_{\text{trim}} - \frac{C_3}{|\alpha_{\max}| + \epsilon} - \frac{C_4}{|\phi_{\max}| + \epsilon} - \frac{C_5}{|\beta_{\max}| + \epsilon} - \frac{C_6}{|r_{\max}| + \epsilon} \\ & - \frac{C_7}{|\theta| + \epsilon} - \frac{C_8}{|\phi_{\text{trim}}| + \epsilon} \end{aligned} \quad (5)$$

subject to various constraints depending on applications. Note that Eq. (5) indicates that p (the roll rate) is to be maximized and α_{\max} in the transient motion, ϕ_{\max} (yaw angle), β_{\max} (sideslip) and r_{\max} (yaw rate) are to be minimized. α_{trim} is calculated as the average angle of attack over the whole time period and may be used to define the limiting angle of attack to be discussed later for the F-16. Specific applications are discussed in the next section.

Two fighter configurations will be used to demonstrate the algorithm, one being an F-5A and the other an F-16. A pitch damper design for the F-5A will be considered first. Control to prevent flight departure will be discussed next. For illustrative purposes, all system gains in the following consideration are assumed constant.

A Pitch Damper Design for an F-5A

The algorithm has been tested and found to work well at different flight conditions to design dampers under multiple input conditions. At low angles of attack, calculated results are found to be consistent with existing systems. To demonstrate this computational tool, consider designing a pitch damper at a Mach number of 0.3 and at an altitude of 10,000 ft. The corresponding α_{trim} is determined to be 11.7 deg. Assume that a damping ratio of 0.65 (ζ_{sp1}) is required in the longitudinal dynamic response of the short-period mode. The optimization problem may then be formulated as follows:

Determine the pitch damper gain constant (K_q) to
minimize the difference in the actual (ζ_{sp}) and desired (ζ_{sp1}) damping ratios; and
 subject to the constraints that

$$0 < \zeta_{sp} < 1$$

$$0 < \omega_{n_{sp}} < 10 \text{ rad/sec.}$$

Limitations on the control system are that
 the pitch rate feedback be limited to 4 deg/sec,
 the elevator deflection limits are +5.5 deg to -17 deg., and
 the elevator actuator rate limits are -26 to +26 deg/sec.

Fig. 1 shows that the pitch damper gain constant to satisfy this design problem is 4.36. The existing system with $K_q = 0.2$ is not adequate to provide a damping ratio of 0.65. Note that during the design process, motions along all axes have been imposed to provide any possible effect of inertial coupling.

Control System to Prevent Yaw Divergence of an F-5A

The second example is to design a control system to prevent yaw divergence of an F-5A during roll maneuver at high angles of attack. The aircraft is placed in a departure condition by a maximum constant elevator deflection to increase the angle of attack, followed by a constant roll control deflection of 2 deg. The optimal control problem is formulated as follows.

Determine the aileron-rudder interconnect gain (K_{ARI}), the side acceleration feedback gain (K_{ay}), and the yaw damper gain (K_r), to
maximize the roll rate, and
minimize the sideslip angle (β), the yaw rate (r), and the change in heading angle (ψ).

In other words, in Eq. (5) the terms associated with α_{trim} and $|\alpha_{max}|$ are not used. Some results for the time variation of motion variables are shown in Fig. 2. It is seen that the present method is effective in satisfying the design objective by reducing both the change in yaw and bank angles.

Control System to Prevent Pitch Departure of an F-16

The aerodynamic data are obtained from ref. 9. Since the F-16 is unstable in pitch, design of a pitch control system is of major concern. The system includes an angle-of-attack and normal-acceleration feedback control. The airplane is first pulled up by applying the full stabilator deflection command (-25°). The objective is to minimize Eq. (5) with

$$\begin{aligned} C_1 &= 0.01, C_2 = 0.03, C_3 = 1, C_4 = 12, C_5 = 0.01 \\ C_6 &= 0.008, C_7 = 1, C_8 = 1 \end{aligned}$$

These weighting factors are chosen so that various terms in Eq. (5) have the same order of magnitude. The design variables are the various gain constants. Note that the α -feedback system is defined such that

$$\delta_e \text{ due to } \alpha\text{-feedback} = K_\alpha \alpha - K_c \quad (6)$$

Two flight conditions are examined, one without imposing a roll maneuver after pull-up and the other with a roll maneuver. Results for the first case are presented in Fig. 3. It is seen that if there is no angle-of-attack limiting system ($K_\alpha = 0, K_c = 0$), the airplane will trim at an angle of attack equal to about 66 deg, which is the deep-stall condition. On the other hand, the limiting system would limit the trim angle of attack to about 25 deg.

For the second case, a roll control of -10 deg is applied between $t = 22$ and 34 sec. Note that roll should induce pitch-up due to inertial coupling. The results shown in Fig. 4 indicate that no departure has occurred and α_{trim} is determined to be 24.7 deg. By changing the initial time at which the roll control is applied, α_{trim} can still be determined to be about 25 deg. Therefore, it may be concluded that maximizing α_{trim} is to define approximately the limiting angle of attack.

CONCLUSIONS

Application of numerical optimization techniques to control system design was demonstrated for F-5A and F-16 configurations at high angles of attack. The methodology accounted for nonlinearities in aerodynamics and dynamics. Specific examples were presented to design control systems to satisfy flying qualities requirements and to prevent flight departure. The results indicated that the present method was effective in satisfying design objectives.

REFERENCES

1. Gilbert, W. P.; Nguyen, L. T.; and Van Gunst, R. W. "Simulator Study of Applications of Automatic Departure-and-Spin Prevention Concepts to Variable-Sweep Fighter Airplane." NASA TM X-2928, 1973.
2. Nguyen, L. T.; Gilbert, W. P.; and Van Gunst, R. W. "Simulator Study of the Departure Resistance of Lightweight Fighter Airplane with Twin Vertical Tails." NASA TM X-3510, 1977.
3. Gilbert, W. P.; Nguyen, L. T.; and Van Gunst, R. W. "Simulator Study of the Effectiveness of an Automatic Control System Designed to Improve the High-Angle-of-Attack Characteristics of a Fighter Airplane." NASA TND-8176, 1976.
4. Gilbert, W. P.; and Libbey, C. E. "Investigation of an Automatic Spin Prevention System for Fighter Airplanes." NASA TND-6670, 1972.
5. Nguyen, L. T.; Gilbert, W. P.; and Ogburn, M. E. "Control System Techniques for Improved Departure/Spin Resistance for Fighter Aircraft." NASA TP-1689, 1989.
6. Linse, D. J.; and Downing, D. R. "The Design and Analysis of a High Angle of Attack Flight Control System." KU-FRL-776-1, The University of Kansas Center for Research, Inc., July 1987.
7. Fan, M. K. H.; Wang, L. S.; Koninckx, J.; and Tits, A. L. "CONSOLE: A CAD Tandem for Optimization-Based Design Interacting with User-Supplied Simulators." Workshop on Computational Aspects in the Control of Flexible Systems, NASA TM-101578, 1988, pp. 89-108.
8. Vanderplaats, G. N. "COPES/ADS--A FORTORN Control Program for Engineering Synthesis Using the ADS Optimization Program." Engineering Design Optimization, Inc., June 1985.
9. Nguyen, L. T., et al. "Simulator Study of Stall/Post-Stall Characteristics of a Fighter Airplane with Relaxed Longitudinal Static Stability." NASA TP-1538, December 1979.

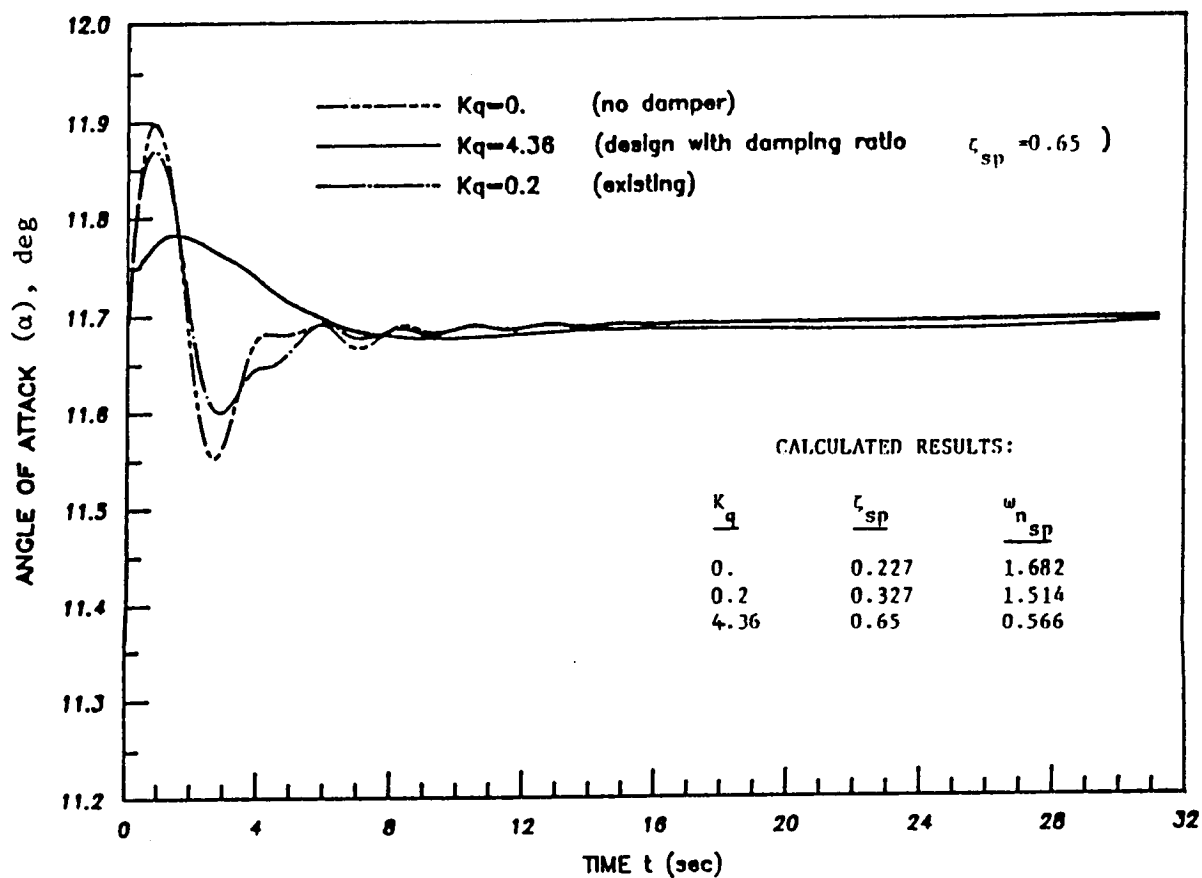


Figure 1 Time History of the Angle of Attack Responding to Impulsive Elevator, Aileron and Rudder Deflections for an F-5A with Pitch Damper ($M=0.3$, $h=10,000$ ft.).

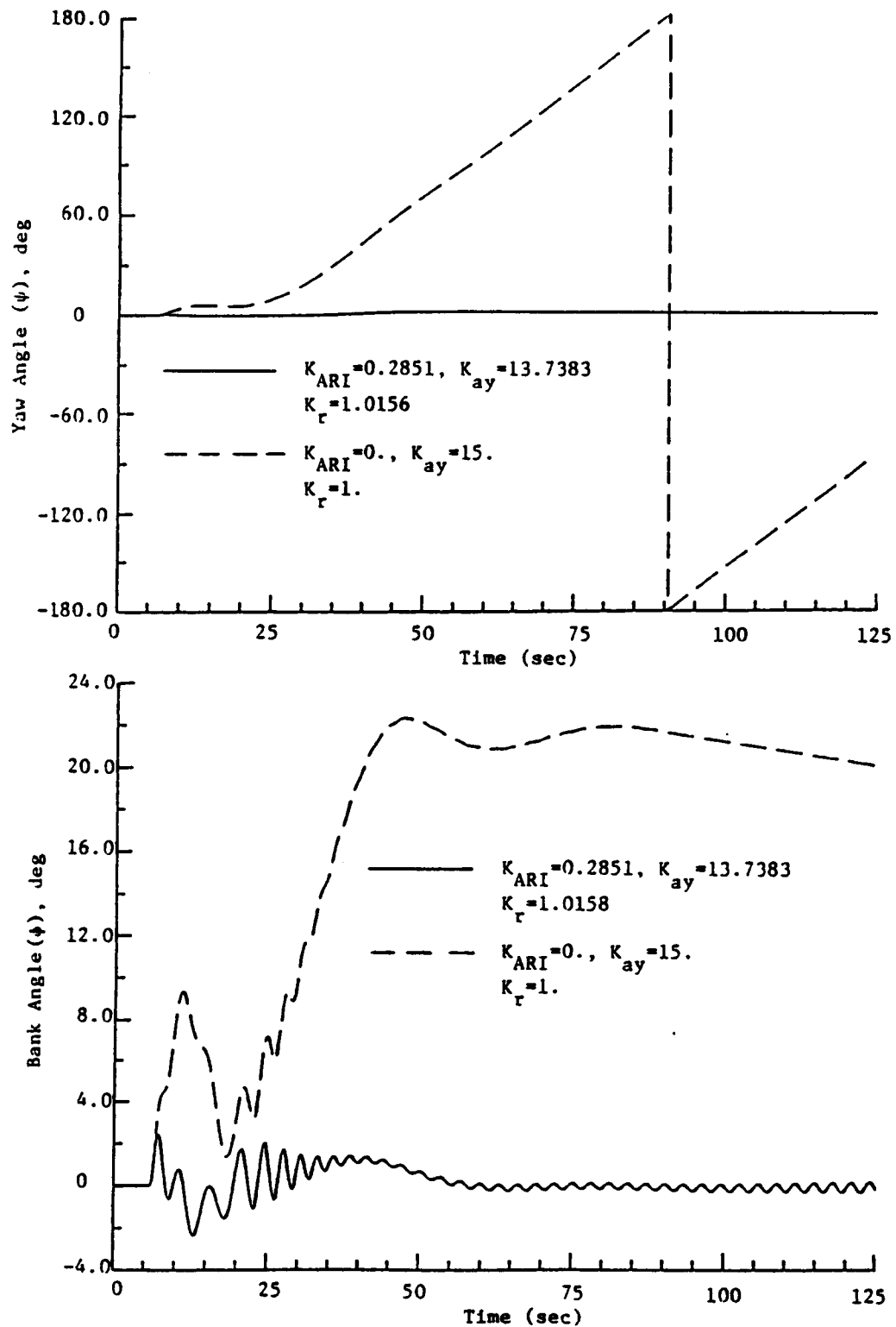


Figure 2 Effect of Control System on Yaw Divergence for an F-5A Configuration at $M = 0.5$ and $h = 10,000$ ft. in a Pull-up and Roll Maneuver

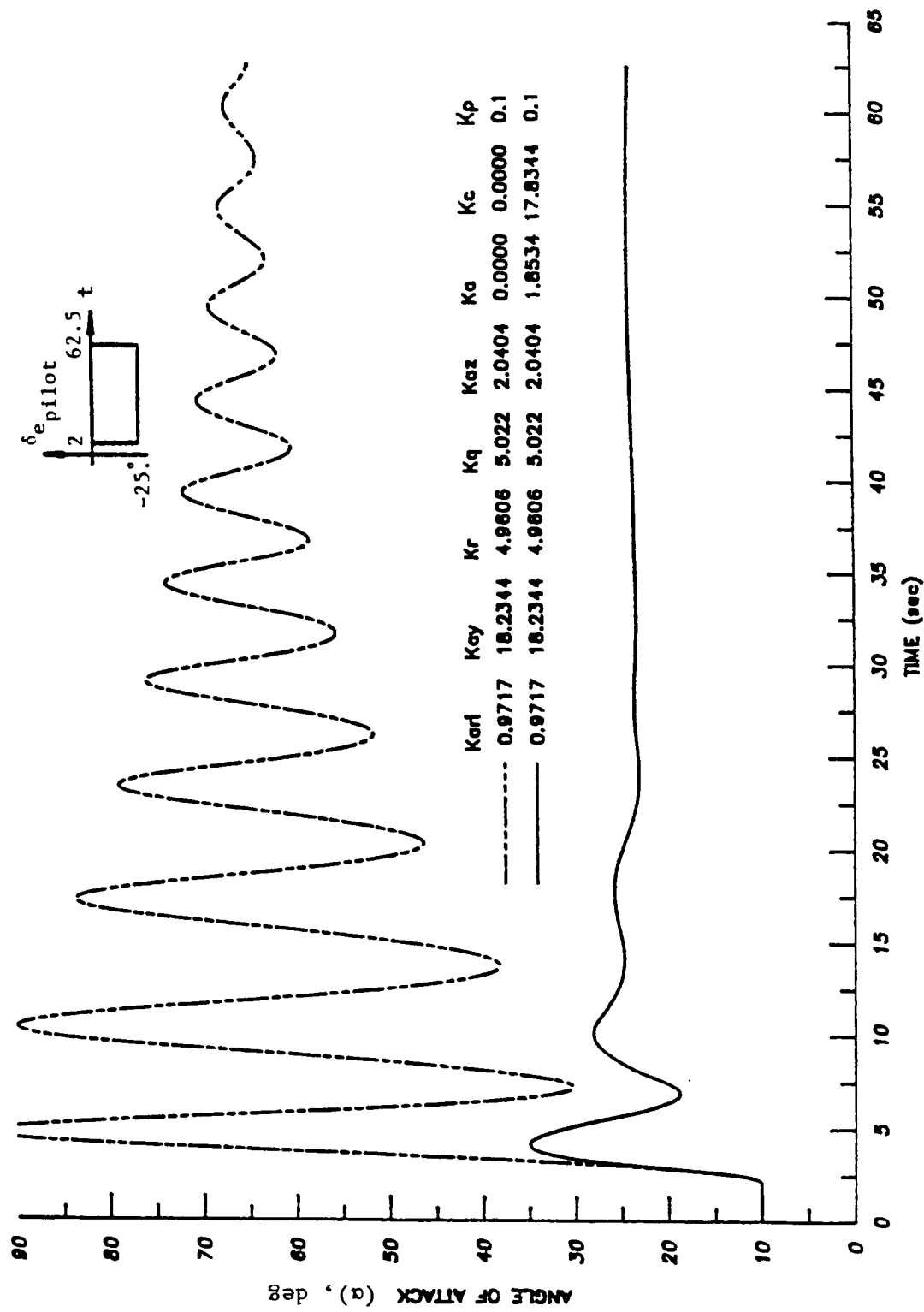


Figure 3 Control System Design to Prevent Pitch Departure of an F-16 Configuration in a Pull-up Maneuver without Roll at $M = 0.5$ and $h = 30,000$ ft.

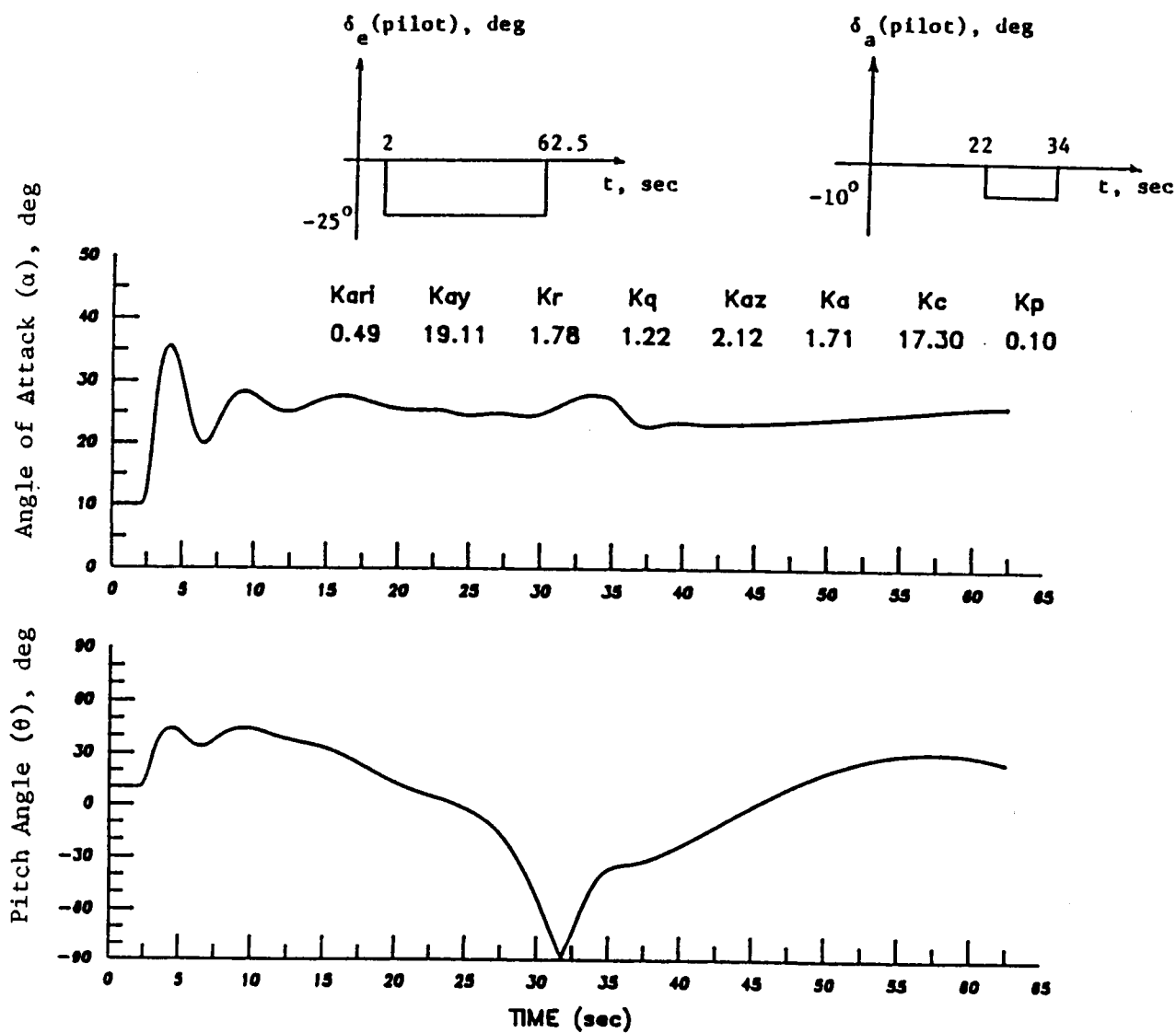


Figure 4 Control System Design to Prevent Pitch Departure of an F-16 Configuration in a Pull-up and Roll Maneuver at $M = 0.5$ and $h=30,000$ ft. Thrust = constant

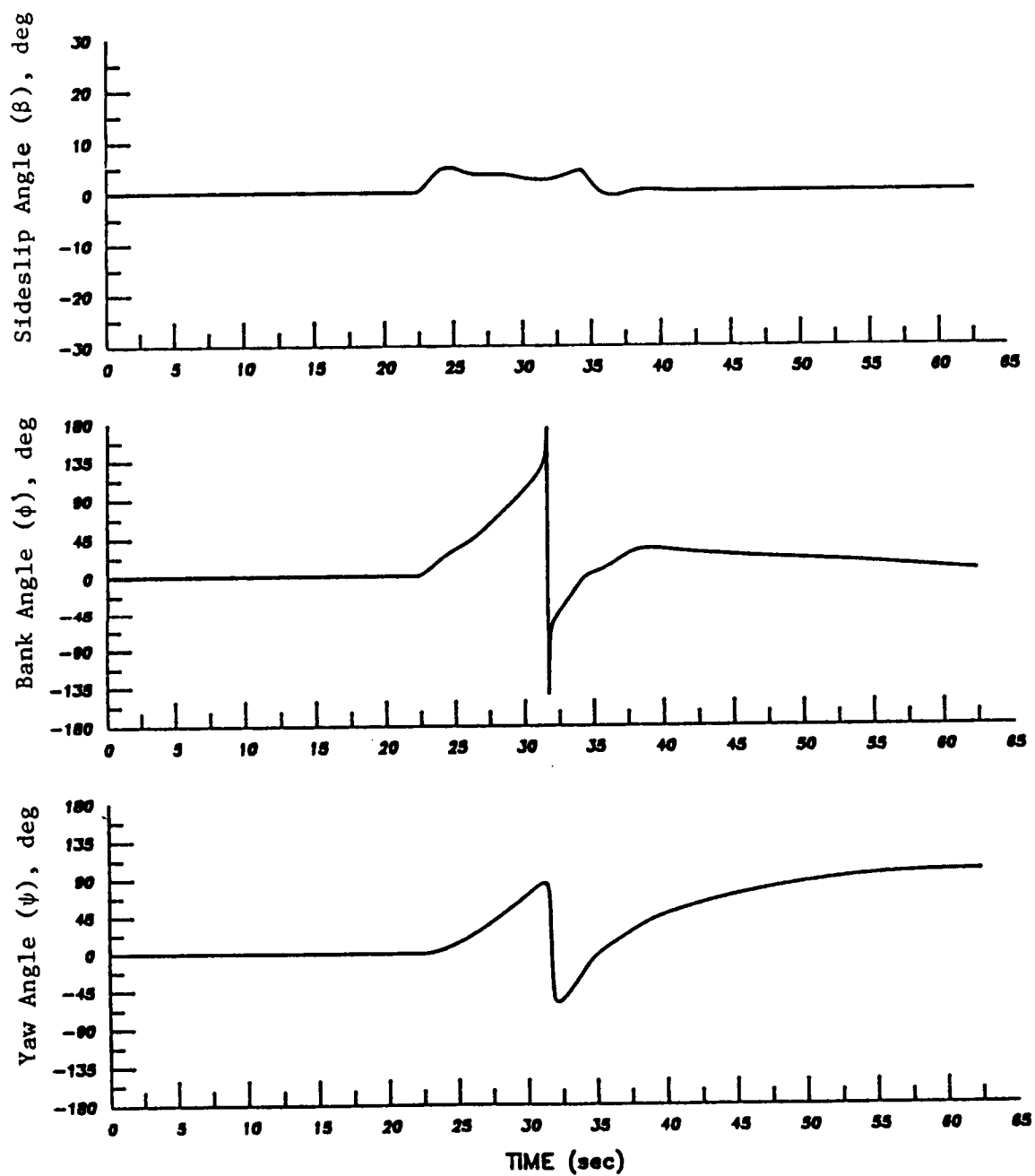


Figure 4 Concluded.